

Fermion mass effects in $e^+e^- \rightarrow 4f$ and $e^+e^- \rightarrow 4f\gamma$ with cuts¹

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Abstract

The fermion mass effects in $e^+e^- \rightarrow 4f$ and in the corresponding bremsstrahlung reactions in the presence of realistic cuts are studied. It is shown that, for some four-fermion final states, the mass effects become sizeable to the extent that they may affect the accuracy of theoretical predictions which is required to be better than 1%.

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1 Introduction

An increasing precision of the measurements of W -boson-pair production at LEP2 and the prospect of improving it at high energy linear colliders requires a corresponding high precision in the Standard Model (SM) theoretical predictions. In order to match the requirements of the final LEP2 data analysis, it is necessary to provide theoretical predictions at a precision better than 1% [1]. The expected experimental precision of future e^+e^- colliders will be probably about 0.1%. Such high precision of the theoretical predictions can only be achieved by including the complete set of electroweak radiative corrections at the one-loop level as well as the leading electroweak logarithms at higher-loops of the Standard Model.

A calculation of the complete set of the SM one-loop virtual corrections to $e^+e^- \rightarrow 4f$, which are reactions of actual interest, is a very tedious task, and despite the fact that some progress in calculating the corrections to $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ was reported in Ref. [2], at present there is no final result available for any of the possible four-fermion final states. Fortunately, a theoretical precision which is satisfactory for most applications in the analysis of the LEP2 data, can be achieved within the double-pole approximation (DPA) [3]. Recently, an interesting complete analysis of the virtual and real photonic corrections in the DPA has been reported in Ref. [4]. In Ref. [4], the DPA has been applied actually only to the non-leading virtual $\mathcal{O}(\alpha)$ corrections while real photonic corrections have been obtained with the full matrix elements of $e^+e^- \rightarrow 4f\gamma$ in the massless fermion limit. A great advantage of the double-pole approximation is that its basic ingredients such as the production of the on-shell W -pairs [5] and W -boson decay [6] have already been calculated at one-loop in the SM. Also the so called non-factorizable virtual corrections are known to have a simple structure and can be found in the literature [7].

Real photon corrections and in particular the hard bremsstrahlung are inherent ingredients of the $\mathcal{O}(\alpha)$ radiative corrections to the four-fermion processes and precise treatment of them is crucial for the ultimate accuracy of theoretical predictions, which for a proper analysis of the final LEP2 data, should possibly match the level of 0.5% [1]. At the moment, there exist several packages which allow one to calculate cross sections of $e^+e^- \rightarrow 4f\gamma$ for any possible final state. They have been compared in Ref. [1]. Three of the codes based on full matrix elements: **WRAP**, **RacoonWW** and **PHEGAS/HELAC** have been subject to tuned comparisons in the approximation of massless fermions in the presence of cuts and they show a very good agreement for the final states considered in Ref. [1]. A complete list of results for total cross sections of all representative processes $e^+e^- \rightarrow 4f\gamma$ can be found in Ref. [8].

The fermion mass effects for $e^+e^- \rightarrow 4f$ and $e^+e^- \rightarrow 4f\gamma$ for different $CC10$ final states have been studied in Ref. [9] for the total cross sections without cuts, except for the photon energy cut. It has been shown that the cross sections of $e^+e^- \rightarrow 4f\gamma$ differ by up to a few per cent for final states including particles with different masses. The natural explanation for it is that the collinear divergences are regularized by different fermion masses in each case and therefore a small change in the fermion mass results in sizeable effects in the total cross sections. One would not expect any numerically sizeable effects of the fermion masses in the presence of angular and invariant mass cuts, as the angles between particles are relatively big then. However, as it will be demonstrated in the following, the massless fermion limit which is usually used in the non-collinear phase space region, may be a source of inaccuracy which may substantially affect the desired accuracy level of 0.5%.

2 The calculation

In the present section we sketch the basics of the calculation. We refer to Refs. [10] and [9] for more details.

The matrix elements of the reactions considered in the present paper are calculated with the helicity amplitude method. Parts of the Feynman graphs which contain a single uncontracted Lorentz index are defined as generalized polarization vectors and used as building blocks of the amplitudes. Particular care is taken of the photon radiation off the external fermion lines. The corresponding fermion propagators are expanded in the light fermion mass analytically in order to avoid numerical cancellations. Fermion masses are kept non zero both in the kinematics and in the matrix elements. Keeping the non zero fermion masses allows for the proper treatment of the collinear photons. Therefore cross sections can be calculated independently of angular cuts and the background from undetected hard photons can be estimated. Moreover, a photon exchange in the t -channel can be handled properly and the Higgs boson effects can be incorporated consistently.

The photon propagator is taken in the Feynman gauge and the propagators of the massive gauge bosons W^\pm and Z^0 , are defined in the unitary gauge. The constant widths Γ_W and Γ_Z are introduced through the complex mass parameters

$$M_V^2 = m_V^2 - im_V\Gamma_V \quad (1)$$

in the propagators. However, the electroweak mixing parameter is kept real, although there is no obstacle to having it complex. This simple prescription preserves the electromagnetic gauge invariance with the non-zero fermion masses, even when the widths Γ_W and Γ_Z are treated as two independent parameters, which has been checked numerically, and for some final states, also analytically.

The constant width prescription violates the $SU(2)$ gauge-symmetry. However, the corresponding numerical effects caused by spoiling the gauge cancellations are in the presence of cuts, practically irrelevant up to the relatively high c.m.s. energy of 10 TeV. This observation relies on the comparison with the results of Ref. [8]. Our results for $e^+e^- \rightarrow u\bar{d}e^-\bar{\nu}_e$, $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ and the corresponding bremsstrahlung reactions, which were calculated in the linear gauge, agree within statistical errors with those of Ref. [8] which were obtained in a nonlinear gauge in the so called complex-mass scheme that preserves the Ward identities.

The phase space integration is performed numerically. The 7 (10) dimensional phase space element of $e^+e^- \rightarrow 4f$ ($e^+e^- \rightarrow 4f\gamma$) is parametrized in several different ways, which are combined in a single multichannel Monte Carlo (MC) integration routine. In the soft photon limit, the photon phase space is integrated analytically.

The most relevant peaks of the matrix elements, e.g., the collinear peaking related to the initial and final state radiation, the $\sim 1/t$ pole caused by the t -channel photon-exchange, the $\sim 1/k$ peaking of the bremsstrahlung photon spectrum, the Breit-Wigner shape of the W^\pm and Z^0 resonances, the $\sim 1/s$ behavior of a light fermion pair production, and the $\sim 1/t$ pole due to the neutrino exchange are mapped away before applying the MC integration routine **VEGAS** [11].

3 Numerical results

In this section, we will present numerical results for several different four-fermion reactions $e^+e^- \rightarrow 4f$ and the corresponding bremsstrahlung reactions.

We define the set of physical parameters, as in Ref. [1], by the gauge boson masses and widths:

$$m_W = 80.35 \text{ GeV}, \quad \Gamma_W = 2.08699 \text{ GeV}, \quad m_Z = 91.1867 \text{ GeV}, \quad \Gamma_Z = 2.49471 \text{ GeV}, \quad (2)$$

by the couplings which are defined in terms of the electroweak mixing parameter $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ and the fine structure constant at two different scales, α_W and α , the latter being used for parametrization of couplings of the external photon,

$$\alpha_W = \sqrt{2}G_\mu m_W^2 \sin^2 \theta_W / \pi, \quad \alpha = 1/137.0359895, \quad (3)$$

with $G_\mu = 1.1663710^{-5} \text{ GeV}^{-2}$.

For the sake of comparison with Ref. [8] we introduce the second set of parameters, originally defined in Ref. [12], which are given again by the gauge boson masses and widths

$$m_W = 80.23 \text{ GeV}, \quad \Gamma_W = 2.0337 \text{ GeV}, \quad m_Z = 91.1888 \text{ GeV}, \quad \Gamma_Z = 2.4974 \text{ GeV}, \quad (4)$$

the single value of the fine structure constant and the electroweak mixing parameter

$$\alpha_W = 1/128.07, \quad \sin^2 \theta_W = \pi / (\sqrt{2} \alpha_W G_\mu m_W^2), \quad (5)$$

with $G_\mu = 1.16639^{-5} \text{ GeV}^{-2}$.

The charged lepton masses are given by

$$m_e = 0.51099906 \text{ MeV}, \quad m_\mu = 105.658389 \text{ MeV}, \quad m_\tau = 1777.05 \text{ MeV} \quad (6)$$

and for the quark masses we take

$$m_u = 5 \text{ MeV}, \quad m_d = 10 \text{ MeV}, \quad m_s = 150 \text{ MeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 5 \text{ GeV}. \quad (7)$$

We define a set of cuts, identical to those of Ref. [8]

$$\begin{aligned} \theta(l, \text{beam}) &> 10^\circ, & \theta(l, l') &> 5^\circ, & \theta(l, q) &> 5^\circ, \\ \theta(\gamma, \text{beam}) &> 1^\circ, & \theta(\gamma, l) &> 5^\circ, & \theta(\gamma, q) &> 5^\circ, \\ E_\gamma &> 0.1 \text{ GeV}, & E_l &> 1 \text{ GeV}, & E_q &> 3 \text{ GeV}, \\ m(q, q') &> 5 \text{ GeV}, & & & & \end{aligned} \quad (8)$$

where l, q, γ , and “beam” denote charged leptons, quarks, photons, and the beam (electrons or positrons), respectively, and $\theta(i, j)$ the angles between the particles i and j in the c.m.s. Furthermore, $m(q, q')$ denotes the invariant mass of a quark pair qq' .

We perform a number of checks. The matrix elements have been checked against **MADGRAPH** [13] and the phase space integrals have been checked against their asymptotic limits obtained analytically. The electromagnetic gauge invariance of the matrix element of the bremsstrahlung process has been checked numerically and for some final states also analytically.

The cut independence of the total bremsstrahlung cross section $\sigma_\gamma = \sigma_s + \sigma_h$ has been tested. σ_s denotes the soft photon contribution to the cross section, which includes photons of energy

$E_\gamma \leq \omega$, and σ_h is the corresponding hard bremsstrahlung cross section for photons of energy $E_\gamma > \omega$. Typical² results are presented in Table 1, where we have used parameters the Eqs. (2), (5) and (6). As the infrared (IR) singular virtual one-loop corrections have not been included, σ_s depends on a small fictitious photon mass m_γ which has been chosen to be $m_\gamma = 10^{-6} \text{ GeV}$. The results in Table 1 are in a sense a measure of the numerical stability of our calculation.

Table 1: Bremsstrahlung cross sections σ_s , σ_h and $\sigma_s + \sigma_h$ in fb for two different photon energy cuts ω and two c.m.s. energies of selected four-fermion reactions. No other kinematical cuts are imposed. The photon mass is $m_\gamma = 10^{-6} \text{ GeV}$.

Final state	ω (GeV)	$\sqrt{s} = 190 \text{ GeV}$			$\sqrt{s} = 500 \text{ GeV}$		
		σ_s	σ_h	$\sigma_s + \sigma_h$	σ_s	σ_h	$\sigma_s + \sigma_h$
$\nu_\tau \tau^+ e^- \bar{\nu}_e \gamma$	10^{-3}	60.60(4)	420.9(8)	481.5	24.01(4)	683.6 (2.2)	707.6
	10^{-1}	258.1(2)	223.8(4)	481.9	305.3(3)	403.4(1.2)	708.7
$\nu_\tau \tau^+ \mu^- \bar{\nu}_\mu \gamma$	10^{-3}	63.26(3)	314.6(4)	377.9	13.74(1)	155.8(3)	169.5
	10^{-1}	210.8(1)	167.2(2)	378.0	73.46(4)	96.0(2)	169.5
$\nu_\mu \bar{\nu}_\mu \tau^- \tau^+ \gamma$	10^{-3}	2.806(1)	13.97(2)	16.78	0.8071(6)	6.193(11)	7.001
	10^{-1}	9.171(4)	7.609(11)	16.78	3.158(2)	3.841(7)	6.999

Our results for the fermion mass effects in $e^+e^- \rightarrow 4f$ and $e^+e^- \rightarrow 4f\gamma$ in the presence of the cuts specified in Eq. (8) are shown for several four-fermion final states in Table 2, where we have used the parameters of Eqs. (4–7). In columns 3 and 4, we list respectively the results of Refs. [14] and [8] which were obtained in the massless fermion limit. The results of our calculation with non-zero fermion masses are shown in column 5. Wherever there is a substantial difference between the results of Refs. [14] and [8] and the present work, we present, in column 5, an additional entry representing the result of ours obtained with the initial fermion masses equal to zero and the final quark and/or charged lepton masses equal to m_e . We do not use an exact zero mass limit for the final state fermions as it is not easy to implement it in our kinematics routine, especially for the bremsstrahlung reactions. However, with the cuts of Eq. (8) the tiny value of m_e should not play a numerically relevant role for the final state particles. With this substitution for the fermion masses, our results agree nicely with those of Refs. [14] and [8], except for $e^+e^- \rightarrow \nu_\tau \bar{\nu}_\tau \mu^- \mu^+ \gamma$, where the difference amounts to a few standard deviations. We do not know the reason for this discrepancy. It may come from the fact that it is not possible to approach the exact zero mass limit in the inverse of the invariant mass of the $\mu^+ \mu^-$ pair and the $\sim 1/s_{\mu^+ \mu^-}$ behaviour of the squared matrix element of $e^+e^- \rightarrow \nu_\tau \bar{\nu}_\tau \mu^- \mu^+ (\gamma)$ has to be mapped away in order to improve a convergence of the phase space integral. Actually EXCALIBUR crashes if there is no lower cut on $s_{\mu^+ \mu^-}$. The lower the cut, the greater the cross section. In our case there is a natural lower cut on $s_{\mu^+ \mu^-}$ of $4m_\mu^2$ or $4m_e^2$ in case of the second entry in column 5. Therefore, we suppose that there has been a tiny cut on the invariant mass of the $\mu^+ \mu^-$ pair imposed in Ref. [8] despite the fact that it was not specified in the paper. For the non-radiative channels also a comparison with KORALW [15] has been performed. Results agree within the Monte Carlo errors.

The final states presented in Table 2 can be classified into three different classes I – III. According to classification of Ref. [16] class I corresponds to the CC20 family, class II to the

²i.e., representatives of the classes I, II and III of processes considered below (see Tab. 2)

CC11 family and class III includes the leptonic processes of the NC32 family. We see almost no mass effect for the tree level four-fermion reactions of class I and II. However, there is a difference of $\sim 1\%$ between cross sections of the radiative reactions $e^+e^- \rightarrow u\bar{d}e^-\bar{\nu}_e\gamma$ and $e^+e^- \rightarrow c\bar{s}e^-\bar{\nu}_e\gamma$ as well as between the results for $e^+e^- \rightarrow u\bar{d}s\bar{c}\gamma$ obtained in the mass limit and with the non zero fermion masses. The effect is even larger ($\sim 2\%$) for $e^+e^- \rightarrow \nu_\tau\tau^+e^-\bar{\nu}_e\gamma$ and $e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$.

Table 2: Mass dependence of cross sections at $\sqrt{s} = 190$ GeV (in fb) for three different classes of final states with the cuts of Eq. (8). The second entries in column 5 correspond to the initial fermion masses equal to zero and the final quark and/or charged lepton masses equal to m_e .

σ (fb)	Final state of $e^+e^- \rightarrow$	Ref. [14] ($m_f = 0$)	Ref. [8]	Present work ($m_f \neq 0$)	
1	2	3	4	5	
I	$u\bar{d}e^-\bar{\nu}_e$	691.5(8)	693.6(3)	693.4(6)	
	$c\bar{s}e^-\bar{\nu}_e$	—	—	693.1(6)	
	$u\bar{d}e^-\bar{\nu}_e\gamma$	—	220.8(4)	220.3(7)	
	$c\bar{s}e^-\bar{\nu}_e\gamma$	—	—	218.2(7)	
	$\nu_\mu\mu^+e^-\bar{\nu}_e$	227.0(3)	227.5(1)	227.5(2)	
	$\nu_\tau\tau^+e^-\bar{\nu}_e$	—	—	227.3(2)	
	$\nu_\mu\mu^+e^-\bar{\nu}_e\gamma$	—	79.1(1)	79.0(3)	
	$\nu_\tau\tau^+e^-\bar{\nu}_e\gamma$	—	—	77.5(2)	
II	$u\bar{d}\mu^-\bar{\nu}_\mu$	667.4(8)	666.7(3)	666.7(4)	
	$u\bar{d}\tau^-\bar{\nu}_\tau$	—	—	666.0(3)	
	$u\bar{d}\mu^-\bar{\nu}_\mu\gamma$	—	214.5(4)	213.8(3)	
	$u\bar{d}\tau^-\bar{\nu}_\tau\gamma$	—	—	209.3(5)	
	$\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$	218.7(3)	218.6(1)	218.3(1)	218.5(1)
	$\nu_\tau\tau^+\mu^-\bar{\nu}_\mu\gamma$	—	76.7(1)	75.1(2)	76.6(2)
	$u\bar{d}s\bar{c}$	—	2015.3(8)	2016(1)	
	$u\bar{d}s\bar{c}\gamma$	—	598(1)	593(2)	598(1)
III	$\tau^-\tau^+\mu^-\mu^+$	—	11.02(1)	9.26(1)	11.03(2)
	$\tau^-\tau^+\mu^-\mu^+\gamma$	—	6.78(3)	5.32(3)	6.62(5)
	$\nu_\tau\bar{\nu}_\tau\mu^-\mu^+$	10.121(40)	10.103(8)	10.05(1)	10.095(10)
	$\nu_\mu\bar{\nu}_\mu\tau^-\tau^+$	—	—	8.529(6)	
	$\nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$	—	4.259(9)	4.18(2)	
	$\nu_\mu\bar{\nu}_\mu\tau^-\tau^+\gamma$	—	—	3.167(7)	
	$\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$	8.224(6)	8.218(2)	8.222(5)	
	$\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu\gamma$	—	1.511(1)	1.510(3)	

The fermion mass effects become more pronounced for the reactions belonging to class III. We observe it here already for the tree level reactions $e^+e^- \rightarrow \tau^-\tau^+\mu^-\mu^+$ and $e^+e^- \rightarrow$

$\nu_\mu \bar{\nu}_\mu \tau^- \tau^+$ where it amounts to about 15% and it becomes even stronger for the corresponding bremsstrahlung reactions. The reason for that is the presence of the virtual photon exchange graphs which introduce the $\sim 1/s$ behavior of the matrix element and the lack of the W^\pm -boson exchange graphs which are relatively insensitive to the fermion masses.

4 Summary

We have studied the fermion mass effects for several channels of $e^+e^- \rightarrow 4f$ and $e^+e^- \rightarrow 4f\gamma$ using the method of calculation elaborated in Refs. [10] and [9]. It has been shown that the fermion mass effects may affect the desired accuracy of 0.5% for the final LEP2 data analysis and the expected precision of 0.1% for the linear collider even in the presence of cuts. Therefore, it seems to be much better not to neglect the fermion masses in calculations intending to match the high precision of the present and future experiments.

The cut independent results presented in Table 1 may serve as tests of MC generators which work with massless fermions especially in the collinear regions of phase space.

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